CBSE CLASS X 2007 MATHEMATICS

SET – II (SOLVED)

SECTION - A

1. If x + a is the GCD of $x^2 - 6x + 5$ and $x^2 - 25$, find the value of a.

Ans.

Let $p(x) = x^2 - 6x + 5$ and $q(x) = x^2 - 25$
Factorising the polynomials p(x) and q(x) we get
$p(x) = x^2 - 6x + 5$
$= x^2 - x - 5x + 5$
$= x(x - 1) - 5(x - 1)$
$= (x - 5)(x - 1)$
$q(x) = x^2 - 25$
$= x^2 - 5^2$
$= (x - 5)(x + 5)$ (by using $a^2 - b^2 = (a - b)(a + b)$)
Therefore GCD of p(x) and q(x) = (x − 5).
But x + a is given as the GCD of p(x) and q(x)
Since GCD of the polynomials is unique.
Therefore x + a = x − 5
$\Rightarrow$ a = -5

2. The mean of the following frequency distribution is 62.8. Find the missing frequency x.

Class	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100	100 – 120
Frequency	5	8	X	12	7	8

Ans.

Class	**Frequency (f_i)**	**Mid-Point (x_i)**	**f_ix_i**
0 – 20	5	10	50
20 – 40	8	30	240
40 – 60	x	50	50x
60 – 80	12	70	840
80 – 100	7	90	630
100 - 120	8	110	880
Total	**40 + x**		**2640 + 50x**

Given that $\overline{x} = 62.8$

Also, $\overline{x} = \dfrac{\sum f_i x_i}{\sum f_i}$

Substituting values, we have,

$\Rightarrow 62.8 = \frac{2640 + 50x}{40 + x}$

$\Rightarrow 2512 + 62.8x = 2640 + 50x$

$\Rightarrow 62.8x - 50x = 2640 - 2512$

$\Rightarrow 12.8x = 128$

$\Rightarrow x = \frac{128 \times 10}{128}$

$\Rightarrow x = 10$

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3. Find the sum of first 18 terms of an A.P. whose n^{th} term is 3 – 2n.

Ans.

Let t_n be the n^{th} term of the AP.

$t_n = 3 - 2n$

Therefore $t_1 = 3 - 2(1)$

$= 3 - 2$

$= 1$

$t_2 = 3 - 2(2)$

$= 3 - 4$

$= -1$

Common difference

Therefore, $d = t_2 - t_1$

$= -1 - 1$

$= -2$

We know that $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

Here n = 18, a = 1

$\therefore S_{18} = \frac{18}{2}\left[2(1) + (18-1)(-2)\right]$

$= a\left[2 + 17(-2)\right]$

$= a\left[2 - 34\right]$

$= a\left[-32\right]$

$= -288$

Hence the sum of first 18 terms of an AP is −288.

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4. A washing machine is available for Rs 13,500 cash or Rs 6,500 as cash down payment followed by three monthly instalments of Rs 2,500 each. Find the rate of interest charged under instalment plan.

Ans.

Cash price of the washing machine = Rs 13,500

Cash down payment = Rs 6,500

Balance due = Rs(13,500 – 6,500) = Rs 7,000

Number of equal instalments = 3

Amount of each instalment = Rs 2,500

Amount paid in instalment scheme = Rs 7,500

Therefore, interest paid in instalment scheme = Rs(7,500 – 7,000)

= Rs 500

Principal for the 1^{st} month = Rs 7,000

Principal for the 2^{nd} month = Rs 4,500
Principal for the 3^{rd} month = Rs 2,000
Total = Rs 13,500
Let the rate of interest be r% per annum

$I = \frac{p \times r \times 1}{1200}$

Then, $I = \frac{13,500 \times r \times 1}{100 \times 12}$

$\Rightarrow 500 = \frac{13,500 \times r \times 1}{100 \times 12}$

$\Rightarrow \frac{500 \times 12}{135} = r$

$\Rightarrow$ r = 44.4%

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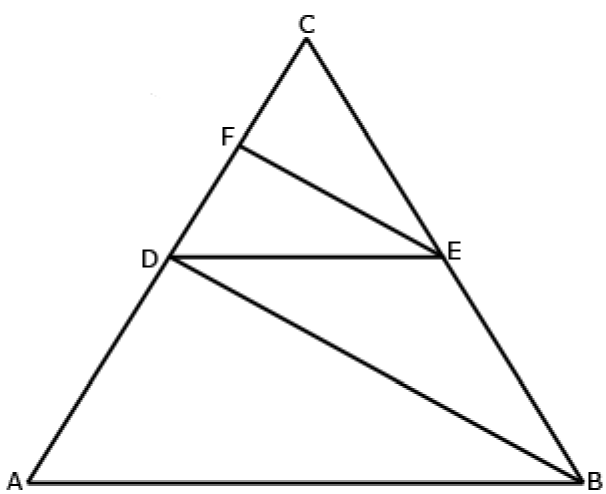
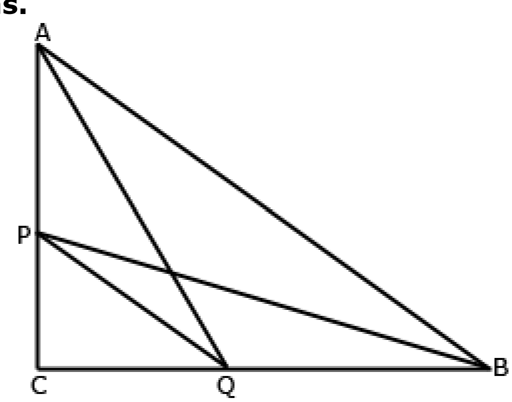
5. P and Q are points on sides CA and CB respectively of ΔABC, right angled at C. Prove that $AQ^2 + BP^2 = AB^2 + PQ^2$

OR

In the figure, DE||AB and FE||DB. Prove that $DC^2 = CF \cdot AC$



Ans.



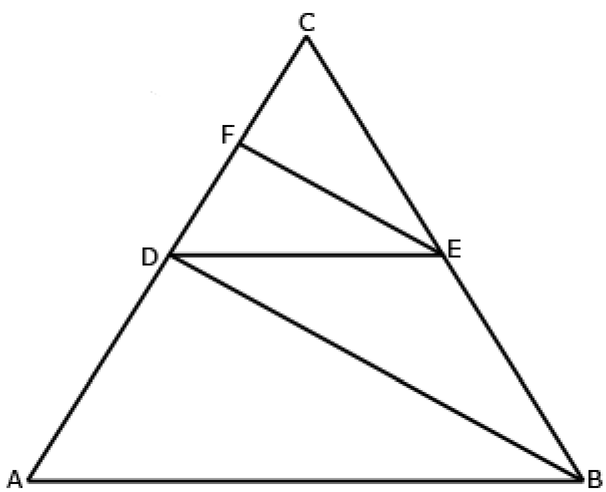
Given: ΔABC is right angled triangle at C.

P and Q are points on sides CA and CB respectively.

To Prove: $AQ^2 + BP^2 = AB^2 + PQ^2$

Proof: In ΔACQ
Using Pythagoras theorem, we have,
$AQ^2 = AC^2 + CQ^2$ (1)
Again applying Pythagoras Theorem in ΔPCB
$BP^2 = PC^2 + BC^2$ (2)
Adding (1) and (2), we get,
$AQ^2 + BP^2 = AC^2 + CQ^2 + PC^2 + BC^2$
$= (AC^2 + BC^2) + (CQ^2 + PC^2)$
$= AB^2 + PQ^2$ [using Pythagoras theorem in ΔABC and ΔPQC, we have, $AC^2 + BC^2 = AB^2$ and $CQ^2 + PC^2 = PQ^2$]
Hence, proved.

OR



Given: DE||AB and FE||DB

To Prove: $DC^2 = CF \cdot AC$

Proof: Since, DE||AB
Therefore, $\frac{CD}{AC} = \frac{CE}{CB}$ (1) (By Basic Proportionality Theorem)
Again since, FE||DB
Therefore, $\frac{CF}{CD} = \frac{CE}{CB}$ (2) (By Basic Proportionality Theorem)
From (1) and (2), we get,
$\frac{CD}{AC} = \frac{CF}{CD}$
$\Rightarrow CD^2 = CF \cdot AC$
Hence, proved.

6. Cards marked with numbers 13, 14, 15, ……, 60 are placed in a box and mixed thoroughly. One card is drawn at random from the box. Find the probability that number on the drawn card is
(i) divisible by 5.
(ii) a number which is a perfect square.

Ans.

We know that probability of an event E is

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Number of total outcomes}}$$

Number of total outcomes = 48.

(i) Let the event E denotes a number divisible by 5

Therefore E = {15, 20, 25, 30, 35, 40, 45, 50, 55, 60}

$$\therefore P(E) = \frac{10}{48} = \frac{5}{24}$$

(ii) Let the event E denotes a number which is a perfect square.

Therefore E = {16, 25, 36, 49}

$$\therefore P(E) = \frac{4}{48} = \frac{1}{12}$$

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7. Solve for x and y:

$$x + \frac{6}{y} = 6$$

$$3x - \frac{8}{y} = 5$$

OR

Solve for x and y:

$$\frac{x+1}{2} + \frac{y-1}{3} = 8$$

$$\frac{x-1}{3} + \frac{y+1}{2} = 9$$

Ans.

$$x + \frac{6}{y} = 6 \qquad (1)$$

$$3x - \frac{8}{y} = 5 \qquad (2)$$

From equation (1), we have,

$$x = 6 - \frac{6}{y}$$

Putting this value of x in (2), we get,

$3\left[6-\frac{6}{y}\right]-\frac{8}{y}=5$

$\Rightarrow 18-\frac{18}{y}-\frac{8}{y}=5$

$\Rightarrow 18-\frac{26}{y}=5$

$\Rightarrow -\frac{26}{y}=5-18$

$\Rightarrow -\frac{26}{y}=-13$

$\Rightarrow 26 = 13y$

$\Rightarrow y = 2$

Therefore, $x=6-\frac{6}{y}$

$\Rightarrow x=6-\frac{6}{2}$

$\Rightarrow x = 6 - 3$

$\Rightarrow x = 3$

Thus, the given system of equations has the solution x = 3 and y = 2.

OR

$\frac{x+1}{2}+\frac{y-1}{3}=8$ (1)

$\frac{x-1}{3}+\frac{y+1}{2}=9$ (2)

Equation (1), can be written as

$\frac{3x+3+2y-2}{6}=8$

$\Rightarrow 3x + 2y + 1 = 48$

$\Rightarrow 3x + 2y = 47$ (3)

Similarly on simplifying equation (2), we get,

$\frac{2x-2+3y+3}{6}=9$

$\Rightarrow 2x + 3y + 1 = 54$

$\Rightarrow 2x + 3y = 53$ (4)

Multiplying (3) by (2) and (4) by (3), we get,

$6x + 4y = 94$ (5)

$6x + 9y = 159$ (6)

Solving (5) and (6)

$-5y = -65$

$\Rightarrow y = 13$

Substituting y = 13 in (3), we get,

x = 7

SECTION – B

8. Solve the following system of equations graphically:
$x - 2y = 4$; $x - y = 3$.

Ans.

The system of equations are given as

$x - 2y = 4$ (i)

$x - y = 3$ (ii)

Equation (i) $\Rightarrow x = 4 + 2y$

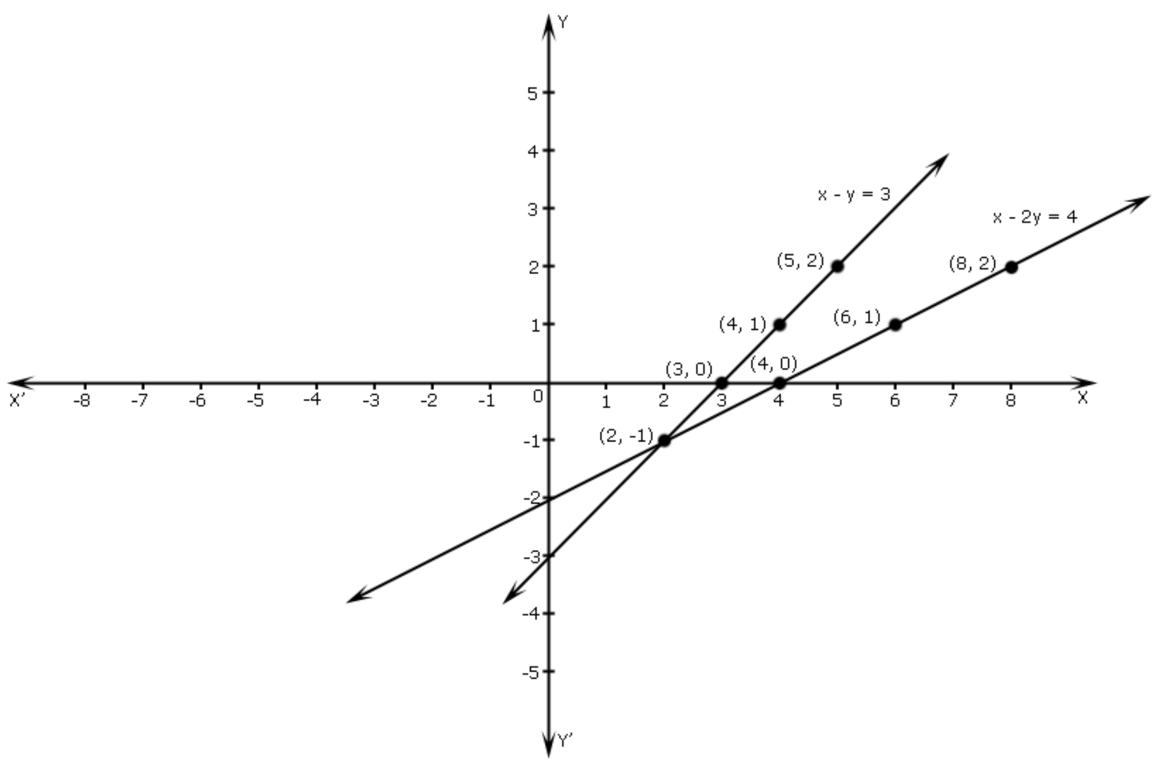
Taking points on the line (i)

x	4	6	8
y	0	1	2

Equation (ii) $\Rightarrow x = 3 + y$

Taking points on the lines (ii)

x	3	4	5
y	0	1	2



Since the two lines are intersecting at the point (2, -1)
Therefore, solution of given system of equations is given by x = 2 and y = -1.

9. Simplify:

$$\frac{x+3}{x^2-1} \div \frac{x^2+2x-3}{x^2-6x-7} + \frac{2(x+2)}{x^2+1-2x}$$

Ans.

Given rational expression is

$$\frac{x+3}{x^2-1} \div \frac{x^2+2x-3}{x^2-6x-7} + \frac{2(x+2)}{x^2+1-2x}$$

$$= \frac{(x+3)}{(x-1)(x+1)} \times \frac{x^2-6x-7}{x^2+2x-3} + \frac{2(x+2)}{x^2-2x+1}$$

$$= \frac{(x+3)}{(x-1)(x+1)} \times \frac{x^2+x-7x-7}{x^2+3x-x-3} + \frac{2(x+2)}{(x-1)^2}$$

$$= \frac{(x+3)}{(x-1)(x+1)} \times \frac{x(x+1)-7(x+1)}{x(x+3)-1(x+3)} + \frac{2(x+2)}{(x-1)^2}$$

$$= \frac{(x+3)}{(x-1)(x+1)} \times \frac{(x-7)(x+1)}{(x-1)(x+3)} + \frac{2(x+2)}{(x-1)^2}$$

$$= \frac{x-7}{(x-1)^2} + \frac{2(x+2)}{(x-1)^2}$$

$$= \frac{x-7+2x+2}{(x-1)^2}$$

$$= \frac{3x-5}{(x-1)^2}$$

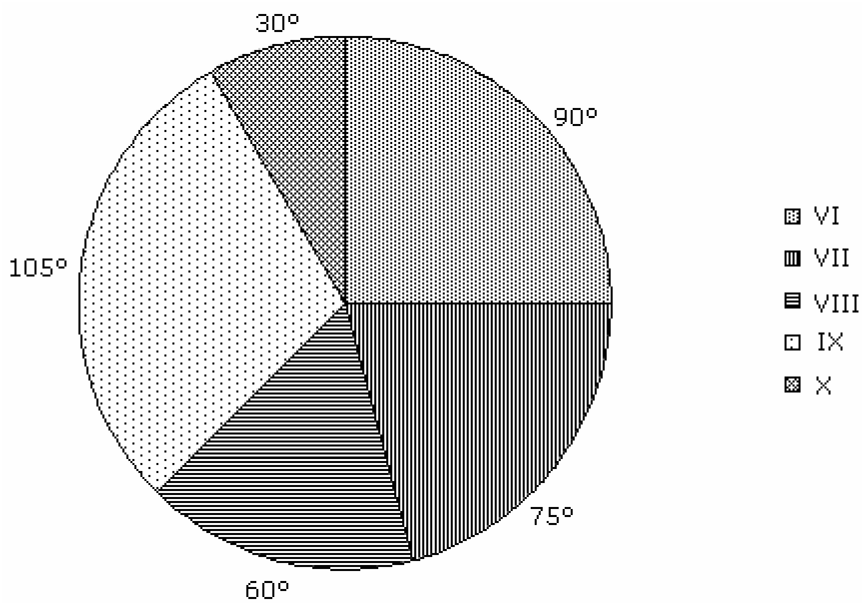
10. The enrolment of a secondary school in different classes is given below:

Class	VI	VII	VIII	IX	X
Enrolment	600	500	400	700	200

Draw a pie chart to represent the above data.

Ans.

Class	Interval	Share as a component of 360°
VI	600	$\frac{600}{2400} \times 360^\circ = 90^\circ$
VII	500	$\frac{500}{2400} \times 360^\circ = 75^\circ$
VIII	400	$\frac{400}{2400} \times 360^\circ = 60^\circ$
IX	700	$\frac{700}{2400} \times 360^\circ = 105^\circ$
X	200	$\frac{200}{2400} \times 360^\circ = 30^\circ$
Total	**2400**	**360°**



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11. Prove that

$$\frac{\cos A}{1-\tan A}+\frac{\sin A}{1-\cot A}=\sin A+\cos A$$

OR

Evaluate without using trigonometric tables:

$$\frac{3\cos 55^\circ}{7\sin 35^\circ}-\frac{4(\cos 70^\circ\ .\ \text{cosec}\,20^\circ)}{7(\tan 5^\circ\ .\ \tan 25^\circ\ .\ \tan 45^\circ\ .\ \tan 65^\circ\ .\ \tan 85^\circ)}$$

Ans.

Given identity is

$$\frac{\cos A}{1-\tan A}+\frac{\sin A}{1-\cot A}=\sin A+\cos A$$

Consider LHS = $\frac{\cos A}{1\text{-}\tan A}+\frac{\sin A}{1-\cot A}$

$$=\frac{\cos^2 A}{\cos A-\sin A}+\frac{\sin^2 A}{\sin A-\cos A} \qquad \left(\text{Using } \tan A=\frac{\sin A}{\cos A},\ \cot A=\frac{\cos A}{\sin A}\right)$$

$$\Rightarrow \frac{\cos^2 A-\sin^2 A}{\cos A-\sin A}=\frac{(\cos A-\sin A)(\cos A+\sin A)}{\cos A-\sin A}$$

= cos A + sin A

= RHS

OR

Consider the expression

$$\frac{3\cos 55^\circ}{7\sin 35^\circ}-\frac{4(\cos 70^\circ\ .\ \text{cosec}\,20^\circ)}{7(\tan 5^\circ\ .\ \tan 25^\circ\ .\ \tan 45^\circ\ .\ \tan 65^\circ\ .\ \tan 85^\circ)}$$

$$= \frac{3\cos(90^\circ - 35^\circ)}{7\sin 35^\circ} - \frac{4(\cos 70^\circ \,.\, \text{cosec}\,(90^\circ - 70^\circ))}{7\tan 5^\circ \,.\, \tan 25^\circ \,.\, 1 \,.\, \tan(90^\circ - 25^\circ)\tan(90^\circ - 85^\circ)}$$

$$= \frac{3\sin 35^\circ}{7\sin 35^\circ} - \frac{4(\cos 70^\circ \,.\, \sec 70^\circ)}{7(\tan 5^\circ \,.\, \cot 5^\circ \,.\, \tan 25^\circ \,.\, 1 \,.\, \cot 25^\circ}$$

[Using $\cos(90° - \theta) = \sin\theta$, $\text{cosec}(90° - \theta) = \sec\theta$, $\tan(90° - \theta) = \cot\theta$)]

$$= \frac{3}{7} - \frac{4}{7}$$

$$= -\frac{1}{7}$$

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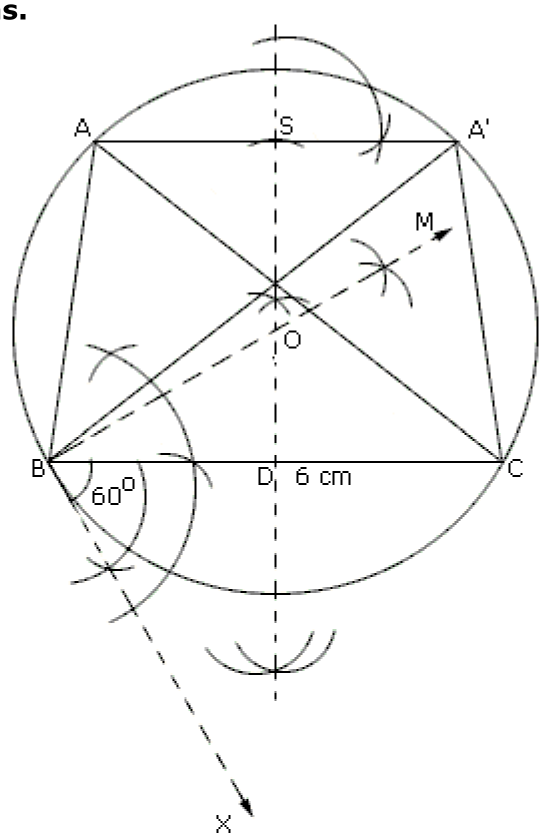
12. Draw a $\triangle PQR$ with base QR = 6 cm, vertical angle P = 60^o and altitude through P to the base is of length 4.5 cm.

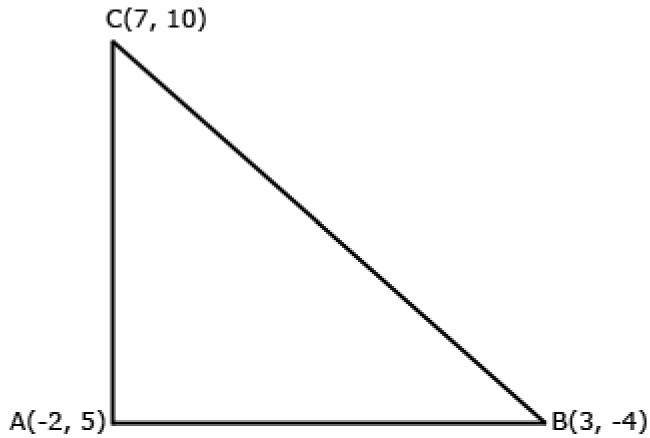
Ans.



13. Show that the points (7, 10), (-2, 5) and (3, -4) are the vertices of an isosceles right triangle.

Ans.

Let the points A(-2, 5), B(3, -4) and C(7, 10) be the vertices of ΔABC



Using distance formula

$AB^2 = (3 + 2)^2 + (-4 - 5)^2 = 25 + 81 = 106$

$BC^2 = (7 - 3)^2 + (10 + 4)^2 = 212$

$AC^2 = (7 + 2)^2 + (10 - 5)^2 = 106$

We see that $BC^2 = AB^2 + AC^2$

$\Rightarrow \angle A = 90°$ (Using Pythagoras Theorem)

Also, AB = AC

Thus, ΔABC is an isosceles right triangle.

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14. A man borrows money from a finance company and has to pay it back in two equal half-yearly instalments of Rs 7,396 each. If the interest is charged by the finance company at the rate of 15% per annum, compounded semi-annually, find the principal and the total interest paid.

Ans.

Amount of the instalment = Rs 7,396

Therefore, rate = $\frac{15}{2}\%$ half yearly

Principal for the 1st instalment will be

Therefore, $P_1 = 7396 \div \left(1 + \frac{15}{200}\right)$

$= 7396 \div \left(\frac{215}{200}\right)$

$= 7396 \times \frac{200}{215}$

= Rs 6,880

Principal for the II instalment will be

$P_2 = 7396 \div \left(1 + \frac{15}{200}\right)^2$

$= 7396 \times \frac{200}{215} \times \frac{200}{215}$

= Rs 6,400

Therefore, principal = $P_1 + P_2$

= Rs 6,880 + 6,400

= Rs 13,280

Total interest paid = Rs 7,396 × 2 − 13,280

= Rs 1,512

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15. A bag contains 6 red balls and some blue balls. If the probability of drawing a blue ball from the bag is twice that of a red ball, find the number of blue balls in the bag.

Ans.

We know that probability of an event E is

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Number of total outcomes}}$$

Let the number of blue balls in the bag be x

and the red balls = 6

Total balls = 6 + x

$\therefore$ P(red ball) $= \frac{6}{(x + 6)}$

and P(blue ball) $= \frac{x}{x + 6}$

According to the question

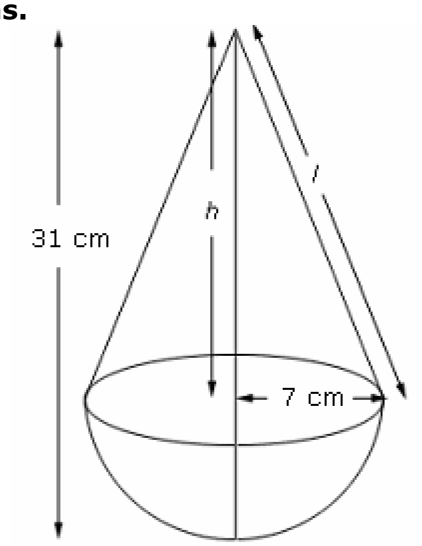
$$\frac{x}{x+6} = 2\left(\frac{6}{x+6}\right)$$

x = 12

Hence, the number of blue balls in the bag is 12.

16. A toy is in the form of a cone mounted on a hemisphere of common base radius 7 cm. The total height of the toy is 31 cm. Find the total surface area of the toy. [Use $\pi = \frac{22}{7}$]

Ans.



Radius of hemisphere = 7 cm
Therefore, radius of base of the cone = 7 cm
Total height of the toy = 31 cm
Height of the conical part = (31 - 7)cm =24 cm

Slant height of the conical part, ℓ

$= \sqrt{h^2 + r^2}$

$= \sqrt{24^2 + 7^2}$

$= \sqrt{576 + 49}$

= 12.5 cm

Therefore, total surface area of the toy
= Surface area of the hemisphere + Surface area of the cone

$= 2\pi r^2 + \pi r l$

$= 2 \times \frac{22}{7} \times 7 \times 7 + \frac{22}{7} \times 7 \times 25$

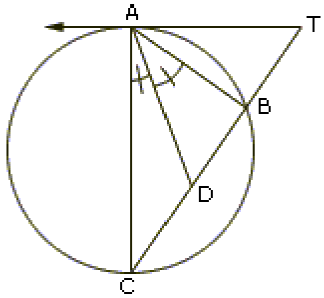
$= 22 \times (14 + 25)$

$= 22 \times 39$

= 858

Therefore, the total surface area of the toy is $858 cm^2$

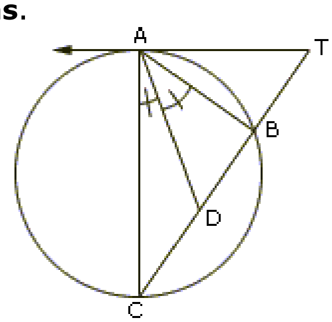
17. In the figure, TA is a tangent to the circle from a point T and TBC is a secant to be circle. If AD is the bisector of ∠CAB, prove that ΔADT is isosceles.



OR

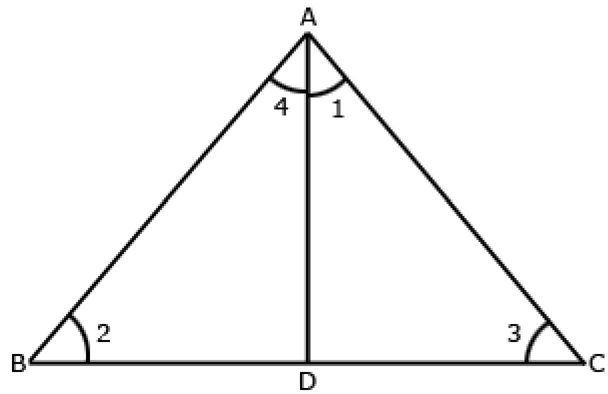
In ΔABC, AD ⊥ BC and AD^2 = BD. DC. Prove that ∠BAC is a right angle.

Ans.



Since, ∠TAB and ∠BCA are angles in the alternate segments of chord AB
⇒ ∠TAB = ∠BCA (i)
Now, AD is the bisector of ∠BAC
Therefore, ∠BAD = ∠CAD (ii)
Now, ∠TAD = ∠TAB + ∠BAD
⇒ ∠TAD = ∠BCA + ∠CAD [using (i) and (ii)]
⇒ ∠TAD = ∠DCA + ∠CAD
⇒ ∠TAD = 180^o - ∠CDA
[since, in ΔCAD, ∠CAD + ∠DCA + ∠CDA = 180^o]
⇒ ∠TAD = ∠TDA [since, ∠CDA + ∠TDA = 180^o by linear pair axiom]
⇒ TD = TA (since, sides opposite to equal angles are equal)
Hence, ΔADT is isosceles

OR



Given: $AD \perp BC$ and $AD^2 = BD \cdot DC$

To prove: $\angle BAC = 90^o$

Proof: In ΔBDA and ΔADC

$\angle ADB = \angle ADC = 90^o$ (Given)

and $\frac{AD}{DC} = \frac{BD}{AD}$ (Given)

Therefore, $\Delta ADC \sim \Delta BDA$ (By SAS similarity rule)

Therefore, $\angle 1 = \angle 2$

and $\angle 1 = \angle 2$ (Angles of similar triangles)

In ΔADC

$\angle 1 + \angle 3 + \angle ADC = 180°$ (Angle sum property)

$\Rightarrow \angle 1 + \angle 3 + 90° = 180°$ (Given $\angle ADC = 90°$)

$\Rightarrow \angle 1 + \angle 3 = 90°$ (1)

Again in ΔABD

$\angle 4 + \angle 2 + \angle ADB = 180°$ (Angle sum property)

$\Rightarrow \angle 4 + \angle 2 = 180° - 90°$ (Given $\angle ADB = 90°$)

$= 90°$ (2)

Adding (1) and (2), we get,

$\angle 1 + \angle 3 + \angle 4 + \angle 2 = 180°$

$\angle 1 + \angle 4 + \angle 4 + \angle 1 = 180°$ (Since, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$)

$\Rightarrow 2\angle 1 + 2\angle 4 = 180°$

$\Rightarrow 2(\angle 1 + \angle 4) = 180°$

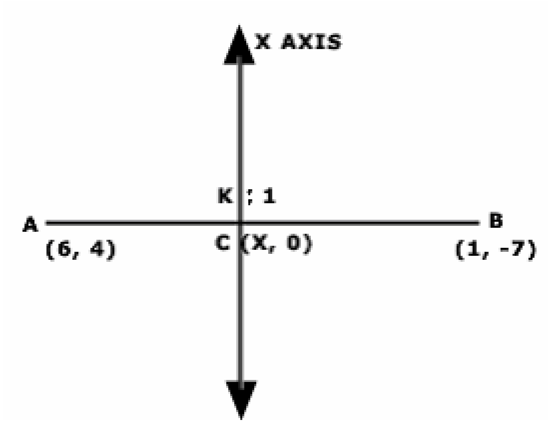
$\Rightarrow \angle 1 + \angle 4 = 90°$

$\Rightarrow \angle BAC = 90°$

Hence, proved.

18. Find the ratio in which the line segment joining the points (6, 4) and (1, -7) is divided by x-axis.

Ans.



Let x axis be intersecting the join of points A(6, 4) and B(1, -7) be at C(x, 0) in the ratio k : 1

Then by using the section formula.

We have

$x = \frac{k+6}{k+1}$

and $y = \frac{-7k+4}{k+1}$

Now, point C has coordinate (x, 0)

So, $\frac{-7k+4}{k+1} = 0$

$\Rightarrow -7k + 4 = 0$

$\Rightarrow k = \frac{4}{7}$

Hence, the required ratio is 4 : 7.

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19. Which term of the AP 3, 15, 27, 39, … will be 120 more than its 64^{th} term?

Ans.

Let t_n be the n^{th} term of the AP which is 120 more than its 64^{th} term.

Therefore, $t_n = 120 + t_{64}$

Here first term a = 3

Common difference d = 15 − 3 = 12.

$\therefore t_n = 120 + a + (64-1)d$

$= 120 + 3 + 63 \times 12$

$= 120 + 3 + 756$

$= 879$

$\therefore t_n = 879$

$\Rightarrow a + (n - 1)d = 879$

$\Rightarrow 3 + (n - 1) \times 12 = 879$

$\Rightarrow 3 + 12n - 12 = 879$

$\Rightarrow -9 + 12n = 879$

$\Rightarrow 12n = 879 + 9$

$\Rightarrow 12n = 888$

$\Rightarrow n = \dfrac{888}{12}$

$\Rightarrow n = 74$

Hence the 74th term will be 120 more than its 64th term.

SECTION – C

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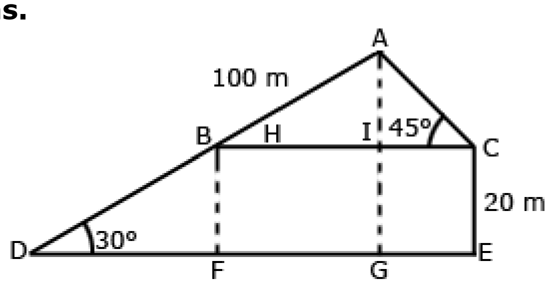
20. A boy standing on a horizontal plane finds a bird flying at a distance of 100 m from him at an elevation of 30°. A girl standing on the roof of 20 m high building, finds the angle of elevation of the same bird to be 45°. Both the boy and the girl are on opposite sides of a bird. Find the distance of bird from the girl.

Ans.



Let the boy is standing at the point D and bird is at point A and CE = 20 m be the building and girl is standing at the point C.

AD = 100 m

AC is distance of bird from the girl.

In ΔAIC

$\dfrac{AI}{AC} = \sin 45^\circ$

$\Rightarrow AI = AC \times \dfrac{1}{\sqrt{2}}$

$\Rightarrow AC = \sqrt{2}\ AI \qquad (1)$

In ΔADG

$\sin 30^\circ = \dfrac{AG}{AD}$

$\Rightarrow \dfrac{1}{2} = \dfrac{AG}{100}$

$\Rightarrow AG = \dfrac{100}{2} = 50$ m

AI = AG – IG

= 50 – 20 (Since, CE = IG = 20 m)

= 30 m

Therefore, AC = $\sqrt{2} \times 30$ (Using (1))

Therefore, AC = 42.426 ~ 42.43 m

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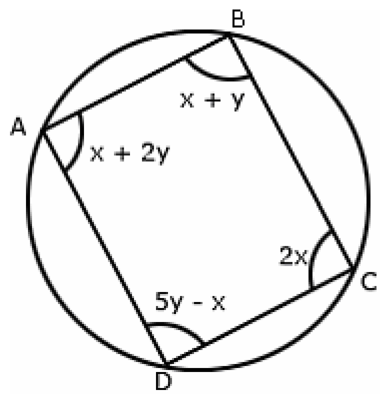
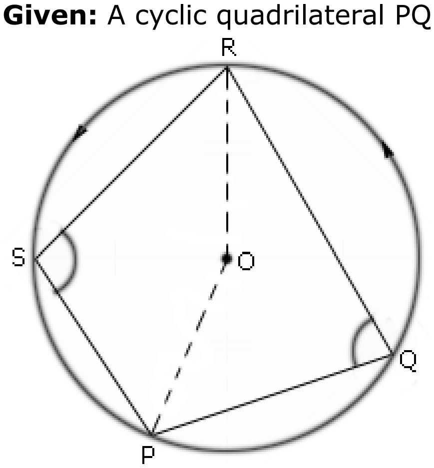
21. Prove that the sum of either pair of opposite angles of a cyclic quadrilateral is 180°. Using the above, find x and y in the figure.



Ans.

Given: A cyclic quadrilateral PQRS and circle C(O, r)



To prove: $\angle P + \angle R = 180^o$

$\angle S + \angle Q = 180^o$

Construction: Join OP and OR

Proof: Since, arc PR subtends $\angle$PSR in its alternate segment,

$\Rightarrow \angle PSR = \frac{1}{2}\angle POR$ (i)

Also, arc RP subtends $\angle$PQR in its alternate segment, therefore,

$\Rightarrow \angle PQR = \frac{1}{2}\angle ROP$ (ii)

Adding (i) and (ii), we get,

$\angle PSR + \angle PQR = \frac{1}{2}(\angle POR + \angle ROP)$

$= \frac{1}{2} \times 360°$

$= 180^o$

$\Rightarrow \angle S + \angle Q = 180^o$

Now, in quadrilateral PQRS, we have,

$\angle P + \angle Q + \angle R + \angle S = 360^o$ (angle sum property of a quadrilateral)

$\Rightarrow \angle P + \angle R + 180^o = 360^o$

$\Rightarrow \angle P + \angle R = 360^o - 180^o$

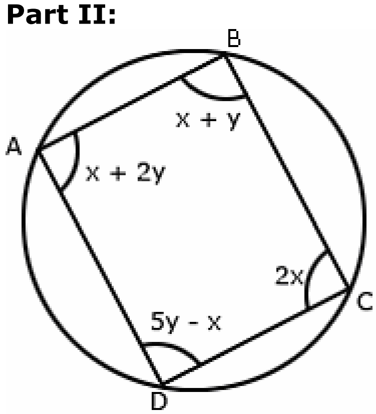
$\Rightarrow \angle P + \angle R = 180^o$

Hence, $\angle P + \angle R = 180^o$

and $\angle S + \angle Q = 360^o$

Hence, proved

Part II:



Here x + y + 5y – x = 180° (Using above Theorem)

$\Rightarrow$ 6y = 180°

$\Rightarrow$ y = 30°

Again x + 2y + 2x = 180° (Using above Theorem)

$\Rightarrow$ 3x + 2y = 180°

$\Rightarrow$ 3x + 2(30°) = 180°

$\Rightarrow$ 3x + 60° = 180°

$\Rightarrow$ 3x = 120°

$\Rightarrow$ x = 40°

Hence, x = 40° and y = 30°.

22. The difference of two numbers is 5 and the difference of their reciprocals is $\frac{1}{10}$. Find the numbers.

OR

By increasing the list price of a book by Rs 10 a person can buy 10 less books for Rs 1,200. Find the original list price of the book.

Ans.

Let one number be x
and other number be y
According to the given condition
x − y = 5 (1)
and $\frac{1}{y} - \frac{1}{x} = \frac{1}{10}$
$\Rightarrow \frac{x - y}{xy} = \frac{1}{10}$
$\Rightarrow \frac{5}{xy} = \frac{1}{10}$ (Using (1))
$\Rightarrow \frac{1}{xy} = \frac{1}{50}$
$\Rightarrow xy = 50$
$\Rightarrow y(y + 5) = 50$ (Using (1))
$\Rightarrow y^2 + 5y - 50 = 0$
$\Rightarrow y^2 + 10y - 5y - 50 = 0$
$\Rightarrow y(y + 10) - 5(y + 10) = 0$
$\Rightarrow (y - 5)(y + 10) = 0$
$\Rightarrow y = 5$ or $y = -10$
If y = 5, then x = 5 + y = 10
If y = -10 then x = 5 + y = 5 − 10 = -5
Hence, the two numbers are 10 and 5 or -5 and -10.

OR

Let the original price of books be Rs x.
and let the number of books be Rs y
Then according to given condition
xy = 1,200 (1)
and (x + 10)(y − 10) = 1200
$\Rightarrow xy - 10x + 10y - 100 = 1200$
$\Rightarrow xy - 10x + 10y = 1300$ (2)
$\Rightarrow 1200 - 10x + 10y = 1300$ (Using (1))
$\Rightarrow -10x + 10y = 100$
$\Rightarrow y - x = 10$
$\Rightarrow \frac{1200}{x} - x = 10$ [Using (1))
$\Rightarrow 1200 - x^2 = 10x$
$\Rightarrow x^2 + 10x - 1200 = 0$
$\Rightarrow x^2 + 40x - 30x - 1200 = 0$

$\Rightarrow x(x + 40) - 30(x + 40) = 0$
$\Rightarrow (x - 30)(x + 40) = 0$
$\Rightarrow x = 30$ or $x = -40$
Reject x = -40 as price cannot be negative.
Therefore, x = 30
Hence, the original price of book is Rs 30.

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23. Ms. Shahnaz earns Rs 35,000 per month (excluding HRA). She donates Rs 30,000 to Prime Minister Relief Fund (100% exemption) and Rs 40,000 to a Charitable Hospital (50% exemption). She contributes Rs 5,000 per month to Provident Fund and Rs 25,000 per annum towards LIC premium. She purchases NSC worth Rs 20,000. She pays Rs 2,300 per month towards income tax for 11 month. Find the amount of income tax she has to pay in 12^{th} month of the year.

Use the following to calculate income tax:

(a) **Savings:** 100% exemption for permissible saving upto Rs 1,00,000

(b) **Rates of income tax for ladies**

Slab	**Income Tax**
(i) Upto Rs 1,35,000	No Tax
(ii) From Rs 1,35,001 to Rs 1,50,000	10% of taxable income exceeding Rs 1,35,000
(iii) From Rs 1,50,001 to Rs 2,50,000	Rs 1,500 + 20% of the amount exceeding Rs 1,50,000
(iv) Rs 2,50,001 and above	Rs 21,500 + 30% of the amount exceeding Rs 2,50,000

(c) **Education Cess:** 2% of income tax payable

Ans.

Gross income of Ms. Shahnaz = Rs 35,000 × 12
= Rs 4,20,000

Donations:
Donation to Prime Minister Relief Fund = Rs 30,000
Donation to Charitable Hospital = Rs 40,000
Exemption = Rs 30,000(100%)
= Rs 20,000(50%)
Exemption = Rs 50,000

Savings:
PF = 5,000 × 12 = Rs 60,000
LIC = Rs 25,000
NSC = Rs 20,000
Total savings = Rs 1,05,000
But saving is subject to maximum of Rs 1,00,000
Therefore, taxable income = Rs(4,20,000 − 50,000 − 1,00,000)
= Rs(4,20,000 − 1,50,000)
= Rs 2,70,000

Income tax = 21,500 + $\frac{30}{100}$(2,70,000 – 2,50,000)

= 21,500 + $\frac{30}{100}$(20,000)

= Rs 21,500 + 6,000

= Rs 2,75,000

Education Cess = $\frac{2}{100} \times 27,500$

= Rs 550

Total tax to be paid = Rs(27,500 + 550)

= Rs 28,050

Income tax paid for 11 months = Rs 2,300 × 11

= Rs 25,300

Therefore, income tax to paid in 12th month of year = Rs(28,050 – 25,300)

= Rs 2,750

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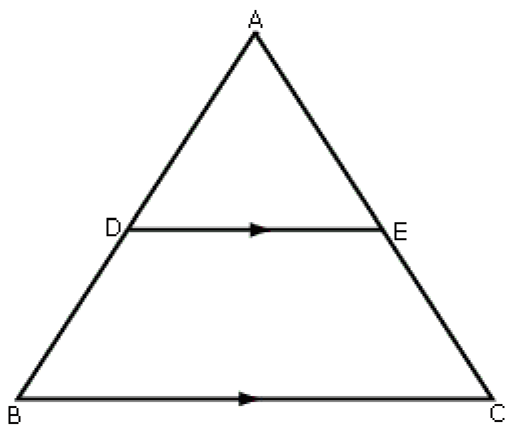
24. If a line is drawn parallel to one side of a triangle, to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio.

Using the above, prove the following:

In the figure, DE||BC and BD = CE. Prove that ABC is an isosceles triangle.

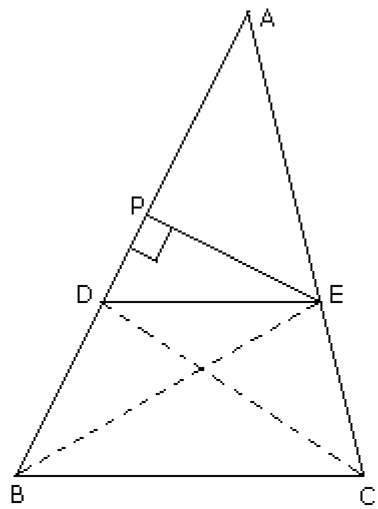


Ans.

Given: ΔABC in which DE||BC

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Join BE, CD and draw EP $\perp$ AB



Proof: ΔDEB and ΔDEC are on the same base DE and between the same parallels DE and BC

Therefore, area ΔDEB = area ΔDEC (i)

$$\frac{\text{area } \Delta ADE}{\text{area } \Delta DEB} = \frac{\frac{1}{2} AD \times EP}{\frac{1}{2} BD \times EP} \quad \left(\text{Area of } \Delta = \frac{1}{2} \text{Base} \times \text{Height}\right)$$

$$= \frac{AD}{BD} \qquad \text{(ii)}$$

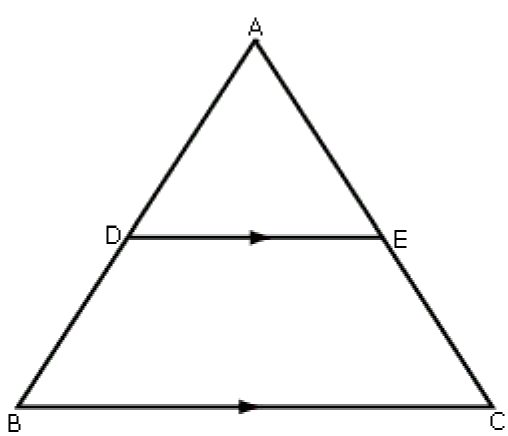
Similarly, $\frac{\text{area } \Delta ADE}{\text{area } \Delta DEC} = \frac{AE}{EC}$

or $\frac{\text{area } \Delta ADE}{\text{area } \Delta DEB} = \frac{AE}{EC}$ (iii) [From (i)]

Using (ii) and (iii), we get**,**

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Part II:



Given: DE||BC and BD = CE

To prove: ΔABC is an isosceles triangle.

Proof: Since, DE||BC

Therefore, $\frac{AD}{DB} = \frac{AE}{EC}$ (By above theorem)

$\frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$

$\Rightarrow \frac{AD + DB}{DB} = \frac{AE + EC}{EC}$

$\Rightarrow \frac{AB}{DB} = \frac{AC}{EC}$

$\Rightarrow$ AB = AC (As BD = CE (given)]

$\Rightarrow \Delta ABC$ is an isosceles triangle.

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25. A sphere, of diameter 12 cm, is dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level in the cylindrical vessel rises by $3\frac{5}{9}$ cm. Find the diameter of the cylindrical vessel.

OR

A solid right circular cone of diameter 14 cm and height 8 cm is melted to form a hollow sphere. If the external diameter of the sphere is 10 cm, find the internal diameter of the sphere.

Ans.

Volume of sphere = $\frac{4}{3}\pi r^3$

Diameter = 12 cm

Therefore, radius = 6 cm

Volume of sphere = $\frac{4}{3} \times \pi \times 6 \times 6 \times 6$ (1)

Let R be radius of cylinder, h be level of water initially and H be the final level

rise in water level = H − h = $\frac{32}{9}$

Increase in volume of the water in cylinder = $\pi R^2(H - h)$

= $\pi \times R \times R \times \frac{32}{9}$ (2)

Increase in volume of water = Volume of sphere

From (1) and (2), we get,

$\frac{4}{3} \times \pi \times 6 \times 6 \times 6 = \pi \times R^2 \times \frac{32}{9}$

$\Rightarrow \frac{4 \times 2 \times 6 \times 6 \times 9}{32} = R^2$

$\Rightarrow R^2 = 3 \times 3 \times 3 \times 3$

$\Rightarrow R = \sqrt{3 \times 3 \times 3 \times 3}$

$\Rightarrow$ R = 9 cm

Therefore, diameter of cylindrical vessel = 2R

= 2(9) = 18 cm

OR

Diameter of cone = 14 cm
Therefore, radius = 7 cm
h = 8 cm

Volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 7 \times 7 \times 8$

External diameter of sphere = 10 cm
Therefore, external radius of sphere r_1 = 5 cm
Let the internal radius of sphere be r_2.
According to the question
Volume of cone = Volume of hollow sphere

$\Rightarrow \frac{1}{3}\pi r^2 h = \frac{4}{3}\pi(r_1^3 - r_2^3)$

$\Rightarrow \frac{1}{3}\pi \times 7 \times 7 \times 8 = \frac{4}{3}\pi(125 - r_2^3)$

$\Rightarrow 7 \times 7 \times 2 = (125 - r_2^3)$

$\Rightarrow 98 - 125 = -r_2^3$

$\Rightarrow -27 = -r_2^3$

$\Rightarrow r_2 = 3$ cm

Hence, the internal radius of the sphere is 3 cm.
Therefore, the internal diameter of the sphere = 3(2)

= 6 cm